GLOBAL OPTIMIZATION APPROACH TO THE BLENDING OF HEAVY FUELS

Onkamon S. N. Ayutthaya[+], Uthaiporn Suriyapraphadilok[+] and Miguel Bagajewicz[*,#]

[+] The Petroleum and Petrochemical College, Chulalongkorn University, 254 Phyathai Road, Patumwan,Bangkok, 10330, Thailand

[*]School of Chemical, Biological and Materials Engineering, University of Oklahoma, Norman, Oklahoma USA 73019

CORRESPONDING AUTHOR: # Miguel Bagajewicz. E-mail address: bagajewicz@ou.edu

KEYWORDS: Fuel Blending, Global Optimization

ABSTRACT

In this article we present a global optimization approach to the problem of blending heavy hydrocarbons, namely Jet Fuel, Diesel and Fuel oil. The problem is nonlinear and is usually solved in industry using some good initial points. To solve it globally, we use RYSIA, a recently developed global optimization methodology based on bound contraction (Faria and Bagajewicz 2011) and compare its performance to Baron and Antigone. The industrial case used proved to be solved in one iteration.

INTRODUCTION

Different fuels that have certain specified properties have to be prepared in a refinery by mixing different fluids that do not meet some of the properties thresholds. The blending problem consists of determining how much of each refinery internal product should be used in a mixture so that the thresholds of certain properties are met. In the case of heavy fuels, we cite: Flash point, Smoke Point, Freezing Point, Conductivity, D90, Density and sulfur and naphthalene contents.

The blending equations used to calculate the properties of the products (Jet Fuel, Diesel and Fuel oil in our case) are non-linear. For example, the flash point of a mixture of fuels, is not the weighted average of the flash points of the ingredients. Thus, the problem requires NLP optimization, which hitherto is mostly done using local optimization solvers.

Blending equations are most of the time nonlinear. To deal with the nonlinearities, industry started to rely on "index-numbers" or "factors" (See Gary and Handwerk, 2007)). The intention at the time of the development of these equations was that the user would be able to use linear relationships in calculations. With the developments of mixing property calculations in the latter part of the last century several nonlinear models were developed. Most of the recent work focuses on the scheduling and distribution aspect of the problem many times included as part of larger planning models and in many cases to treat uncertainty (Moro, Zanin et al. 1998, Jia and Ierapetritou 2003, Mendez, Grossmann et al. 2006, Chen and Wang 2010, Pongsakdi et al., 2006, Lakkhanawat and Bagajewicz, 2008). Models that deal exclusively with the blending problem dissociated from the planning scheduling and distribution issues are not that common. Murty and Rao (2004) presented a model to obtain the minimum cost of blended gasoline based on ANN (artificial neural network) which was done to predict the octane number. It is also very well known that the blending problem is related to the pooling problem (Meyer and Floudas 2006, Faria and

Bagajewicz 2012). Several scheduling models have been also presented by Ierapetritou and Floudas (1998a,b), Ierapetritou et al. (1999), Misener et al., 2010.

In this article, we explore the use of RYSIA, our bound contraction procedure for global optimization (Faria and Bagajewicz 2011). The paper is organized as follows: We present the nonlinear model first. We discuss the bound contraction strategy next, including the lifting discretizations and the uneven interval size bound contraction procedure. We then describe the linearized relaxed model neede by RYSIA and present results.

MODEL FORMULATION

The blending problem can be represented as in Figure 1. Several intermediate products can be used to feed each blender for each product.

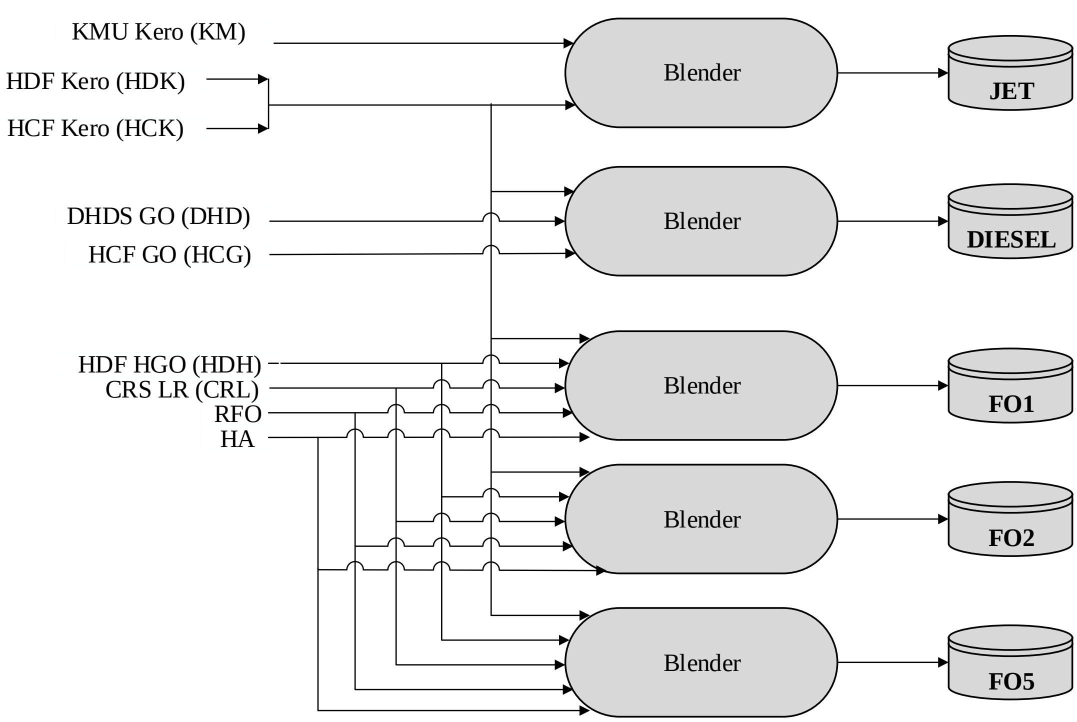

Figure 1. Typical Blending scheme of jet, diesel and fuel oils.

The objective of the blending problem is $PROFIT$. Thus, with the price of products, pr_p, and cost of intermediate streams, ct_s, we have

$$\max PROFIT = \sum_p (pr_p \ WT_p) - \sum_{s,p} (ct_s \ W_{s,p}) \tag{1}$$

where WT_p is the flow of product p and $W_{s,p}$ is the flow of raw intermediate ingredient s to the blender of product p. The constraints are:

Balance equations: We first define the volume of product blend (vT_p) as follows:

$$vT_p = \sum_s V_{s,p} \qquad \forall p \tag{2}$$

where $V_{s,p}$ is the volume of feed s used in the product p blender. The assumption therfore is that the mixing is ideal (no change of volume upon mixing). Density links $W_{s,p}$ and $V_{s,p}$ as follows:

$$den_s \times V_{s,p} = W_{s,p} \times 1{,}000 \qquad \forall s,p \tag{3}$$

Density also links vT_p and WT_p as follows

$$DEN_p \times vT_p = WT_p \times 1{,}000 \qquad \forall p \tag{4}$$

In turn, the total weight of the product WT_p is

$$WT_p = \sum_s W_{s,p} \qquad \forall s,p \tag{5}$$

*Capacity Constraints:*The capacity of components are restricted to a maximum capacity.

$$W_{s,p} \le cp_s \qquad \forall s,p \tag{6}$$

The component properties such as density ($dens_s$), sulfur content ($suls_s$), naphthalene content ($naps_s$), conductivity ($conds_s$), flash point (fps_s), smoke point (sps_s), freezing point ($frzs_s$), viscosity ($viss_s$) and D90 ($d90s_s$) are subject to box constraints:

$$propmin_p \leq PROP_p \leq propmax_p \qquad \forall p \qquad (7)$$

Linear index-based properties: Some properties are calculated using indices : flash point index ($fpis_s$), smoke point index ($spis_s$), freezing point index ($frzis_s$), viscosity index ($v50s_s$) and D90 index ($d90is_s$). The calculation of product properties that can be accomplished in two ways volumetric indices and weight indices:

$$PROP_p \times vT_p = \sum_s (props_s \times V_{s,p}) \qquad \forall p \qquad (8)$$

$$PROP_p \times WT_p = \sum_s (props_s \times W_{s,p}) \qquad \forall p \qquad (9)$$

The following properties are blended using volumetric indices: Naphthalene content (NAP_p), conductivity ($COND_p$), density (DEN_p), flash point (FPI_p), smoke point (SPI_p), Freezing points ($FRZI_p$), D90% points ($D90I_p$) and viscosity $(V50_p)$ for some blends. The corresponding indices are therefore parameters.

The following properties are blended using volumetric indices: Sulphur content (SUL_p), and viscosity $(V50_p)$ for some blends.

Non-linear constraints: The flash point (FP_p) and flash point index FPI_p are related as follows:

$$FP_p = \frac{(a - \ln(FPI_p))}{b} \qquad \forall p \qquad (10)$$

where the flash point is subject to a minimum constraint:

$$Fpmin_p \leq FP_p \qquad \forall p \qquad (11)$$

The smoke point (SP_p) subject to a minimum value and the smoke point index SPI_p are related as follows:

$$SP_p = \frac{1}{SPI_p} \qquad \forall p \qquad (12)$$

$$Spmin_p \leq SP_p \qquad \forall p \qquad (13)$$

The freezing point (FRZ_p) subject to a maximum value and the freezing point index $SFRZI_p$ are related as follows:

$$FRZ_p = \left(-\frac{c}{FRZI_p}\right) + d \qquad \forall p \qquad (14)$$

$$FRZmax_p \geq FRZ_p \qquad \forall p \qquad (15)$$

The distillation point temperature ($D90_p$) subject to a maximum and the distillation point index $D90I_p$ are related as follows:

$$D90_p = -\ln(-D90I_p)/d90coeff \qquad \forall p \qquad (16)$$

$$D90max_p \geq D90_p \qquad \forall p \qquad (17)$$

Viscosity (VIS_p) subject to a minimum and maximum and the viscosity index ($V50_p$) are related as follows:

$$VIS_p = 10^{10^{(V50p - e - (f \times \log\left(\frac{temp_p + 273}{50+273}\right)))/g}} - h \qquad \forall p \qquad (18)$$

$$Vismin_p \leq VIS_p \leq Vismax_p \qquad \forall p \qquad (19)$$

In this equation $temp_p$ is the temperature of product p.

GLOBAL OPTIMIZATION

RYSIA is used here to perform the global optimization and it is compared to BARON and ANTIGONE. RYSIA is based on a procedure that uses a combination of lower bound for maximization, which in this case is the original NLP model and a Relaxed model that is linear as an upper bound. This relaxation will be shown in the next section. With the use of these two models, the gap between them is tightened using a bound contraction procedure, without resorting necessarily to branch and bound methods.

The global optimization strategy is described briefly next:

(1) The MILP upper bound model is created by linear relaxation and partitioning the variables making the variables float in side intervals defined by this partition (see below). The solution of this model is used as the initial points to NLP lower bound model.

(2) The solution of the MILP upper bound model provides values of variables that are in specific intervals. These values are used as initial points for the lower bound NLP model.

(3) Then each variable that has been partitioned is chosen one at the time to run the restricted MILP model, which consists of forbidding the interval where the original MILP solution lies. If the solution is lower than the lower bound, all the intervals, except the forbidden one are eliminated and the partition of this variable is performed again.

(4) After all variables have been tested for contraction the lower bound is run again to update its value and the upper bound model is run again and step (3) is started again.

UPPER BOUND

The upper bound model is obtained by using all linear constraints and of relaxing the nonlinear values: The quadratic constraints relaxation is described in by (Faria and Bagajewicz 2011, Faria and Bagajewicz 2012). Nonlinear term relaxation was introduced by (Faria and Bagajewicz 2012, Faria et al 2012 and Kim et al, 2016).

We start with defining a new variable:

$$LFPI_p = \log_{10} FPI_p \qquad \forall p \qquad (20)$$

We now rewrite equation (10) as follows:

$$LFPI_p = a - \left(b \times FP_p\right) \qquad \forall p \qquad (21)$$

We now create partitions for FPI_p and $LFPI_p$ using a set of intervals f and binary variables $yFPI_f$ as follows:

$$\sum_f \left(yFPI_f \times fpiat_f\right) \leq FPI_p \leq \sum_f \left(yFPI_f \times fpiat_{f+1}\right) \qquad \forall p \qquad (22)$$

$$\sum_f \left(yFPI_f \times lfpiat_f\right) \leq LFPI_p \leq \sum_f \left(yFPI_f \times lfpiat_{f+1}\right) \qquad \forall p \qquad (23)$$

and we force the solution to be in one and only one interval.

$$\sum_f yFPI_f = 1 \qquad (24)$$

The same concept is applied for $D90I_p$. We first write:

$$LD90I_p = \ln(D90I_p) \qquad \forall p \qquad (25)$$

and therefore (16) becomes:

$$D90_p = -\left(\frac{1}{d90coeff}\right) \times LD90I_p \qquad \forall p \qquad (26)$$

which leads to:

$$\sum_f \left(yD90I_f \times d90iat_f\right) \leq D90I_p \leq \sum_f \left(yD90I_f \times d90iat_{f+1}\right) \qquad \forall p \qquad (27)$$

$$\sum_f \left(yD90I_f \times ld90iat_f\right) \leq LD90I_p \leq \sum_f \left(yD90I_f \times ld90iat_{f+1}\right) \qquad \forall p \qquad (28)$$

$$\sum_f \left(yD90I_f\right) = 1 \quad (29)$$

For viscosity we write:

$$\sum_f\left(yV50_f \times v50at_f\right) \leq V50_p \leq \sum_f\left(yV50_f \times v50at_{f+1}\right) \quad \forall p \quad (30)$$

$$\sum_f\left(yV50_f \times visat_f\right) \leq VIS_p \leq \sum_f\left(yV50_f \times visat_{f+1}\right) \quad \forall p \quad (31)$$

$$\sum_f \left(yV50_f\right) = 1 \quad (32)$$

where $visat_f = 10^{10^{(v50at_f - e - (f \times \log\left(\frac{temp_p+273}{50+273}\right)))/g}} - h.$

For the product of the blended sulfur content, SUL_p, and the amount of product p in weight, WT_p, we define

$$ZSUL_p = SUL_p \times WT_p \quad \forall p \quad (33)$$

and then we write:

$$\sum_f\left(ySUL_f \times sulat_f\right) \leq SUL_p \leq \sum_f \left(ySUL_f \times sulat_{f+1}\right) \quad \forall p \quad (34)$$

$$\sum_f\left(WWT_f \times sulat_f\right) \leq ZSUL_p \leq \sum_f \left(WWT_f \times sulat_{f+1}\right) \quad \forall p \quad (35)$$

$$\sum_f \left(ySUL_f\right) = 1 \quad (36)$$

In turn, tThe values of WWT_f (a product of a continuous variable and a binary variable) can be obtained from the equations below.

$$WWT_f - OMEGA \times ySUL_f \leq 0 \quad \forall f \quad (37)$$

$$\left(WT_p - WWT_f\right) - OMEGA \times \left(1 - ySUL_f\right) \leq 0 \quad \forall p, f \quad (38)$$

$$\left(WT_p - WWT_f\right) \geq 0 \quad \forall p, f \quad (39)$$

For bilinearities containing viscosity, smoke point and freezing point, we define:

$$ZV50_p = V50_p \times WT_p \quad \forall p \quad (48)$$

$$WSPI_f = ySP_f \times SPI_p \quad \forall p, f \quad (58)$$

A positive variable, $WV50_f$, is introduced as a product of yWT_f and $V50_p$.

$$WV50_f = yWT_f \times V50_p \qquad \forall p, f \quad (49)$$

where yWT_f are binary variables. WT_p is partitioned into $f-1$ intervals and each interval of WT_p starts with a value $wtat_f$. Hence, WT_p is substituted by its discrete bounds as follows:

$$\sum_f \left(yWT_f \times wtat_f\right) \leq WT_p \leq \sum_f \left(yWT_f \times wtat_{f+1}\right) \qquad \forall p \quad (50)$$

$ZV50_p$ is then bounded to correspond to the same interval in the following equation.

$$\sum_f \left(WV50_f \times wtat_f\right) \leq ZV50_p \leq \sum_f \left(WV50_f \times wtat_{f+1}\right) \qquad \forall p \quad (51)$$

The values of $WV50_f$ can be obtained from the equations below.

$$WV50_f - OMEGA \times yWT_f \leq 0 \qquad \forall f \quad (52)$$

$$\left(V50_p - WV50_f\right) - OMEGA \times \left(1 - yWT_f\right) \leq 0 \qquad \forall p, f \quad (53)$$

$$\left(V50_p - WV50_f\right) \geq 0 \qquad \forall p, f \quad (54)$$

Similarly, the summation of all yWT_f must be equal to one to warrant that only the interval that contains solution is selected.

$$\sum_f \left(yWT_f\right) = 1 \quad (55)$$

For this $V50$ index, since the partitioning variables, i.e. WT_p, are different than the bound contract variables, i.e. $V50_p$, a similar partioning procedure is performed to $V50_p$.

$$\sum_f \left(yV50_f \times v50at_f\right) \leq V50_p \leq \sum_f \left(yV50_f \times v50at_{f+1}\right) \qquad \forall p \quad (56)$$

$$\sum_f \left(yV50_f\right) = 1 \quad (57)$$

Smoke point

The smoke point index is equal to the reciprocal of the smoke point in °C and it can be rearranged to. $SPI_p \times SP_p = 1$. A positive variable, $WSPI_f$, is introduced as a product of ySP_f and SPI_p.

$$WSPI_f = ySP_f \times SPI_p \qquad \forall p, f \quad (58)$$

The same partitioning procedure is performed to SP_p and the product of SPI_p and SP_p or in this specific case is the value of unity.

$$\sum_f \left(WSPI_f \times spat_f\right) \leq 1 \leq \sum_f \left(WSPI_f \times spat_{f+1}\right) \quad (59)$$

$$\sum_f \left(ySP_f \times spat_f\right) \leq SP_p \leq \sum_f \left(ySP_f \times spat_{f+1}\right) \qquad \forall p \quad (60)$$

where $spat_f$ is the starting value of each interval of the variable SP_p and ySP_f are binary variables whose summation is equal to one.

$$\sum_f \left(ySP_f\right) = 1 \quad (61)$$

$WSPI_f$ is calculated by the following equations:

$$WSPI_f - OMEGA \times ySP_f \leq 1 \qquad \forall f \quad (62)$$

$$\left(SPI_p - WSPI_f\right) - OMEGA \times (1 - ySP_f) \leq 0 \qquad \forall p, f \quad (63)$$

$$\left(SPI_p - WSPI_f\right) \geq 0 \qquad \forall p, f \quad (64)$$

Freezing Point

Freezing point is presented in a bilinear term as follows:

$$ZFRZ_p = FRZ_p \times FRZI_p \qquad \forall p \quad (65)$$

Substituting in (26)

$$ZFRZ_p = d + \left(c \times FRZI_p\right) \qquad \forall p \quad (66)$$

and

$$WFRZ_f = yFRZ_f \times FRZI_p \qquad \forall p, f \quad (67)$$

$$\sum_f \left(WFRZ_f \times frzat_f\right) \leq ZFRZ_p \leq \sum_f \left(WFRZ_f \times frzat_{f+1}\right) \qquad \forall p \quad (68)$$

For this $V50$ index, since the partitioning variables, i.e. WT_p, are different than the bound contract variables, i.e. $V50_p$, a similar portioning procedure is performed to $V50_p$.

$$\sum_f (yV50_f \times v50at_f) \leq V50_p \leq \sum_f (yV50_f \times v50at_{f+1}) \qquad \forall p \quad (56)$$

$$\sum_f (yV50_f) = 1 \qquad (57)$$

EXAMPLE

To test the model we used jet fuel, diesel, and fuel oil FO1, FO2 and FO5. All blending components for each final product are shown in Table 1.

Table 1. The intermediate streams of each product

Blending components	**Product**
HDF Kero, HCF Kero, KMU Kero	Jet fuel
HDF Kero, HCF Kero, DHDS GO, HCF GO	Diesel
HDF Kero, HCF Kero, RFO, CRS LR, HDF HGO, HA	Any types of Fuel oil

Table 2 shows prices per ton and the final products quantities of jet fuel, diesel and fuel oil No.1, No.2 and No.5, respectively. Table 3 tabulates costs per ton and the capacities of each intermediate streams. Since this work focus on the reformulation of bilinear terms and quadratic blending equations to linear inequality equations, all product prices and stream costs are set as parameters.

Table 2. Product Prices and Demands

Products	Price ($/ton)	Price ($/barrel)	Demands (m^3)
Jet	1,005.4	148.9	17,000
Diesel	935.2	139.8	15,000
Fuel oil No.1	622.8	97.0	5,500
Fuel oil No.2	622.8	97.0	3,500
Fuel oil No.5	622.8	97.0	4,500

Table3. Costs and Capacities

Streams	Cost ($/ton)	Cost ($/barrel)	Capacities (tons)
KMU Kero	1005.4	148.9	10,000
HDF Kero	1005.4	148.9	10,000
HCF Kero	1005.4	148.9	10,000
DHDS GO	935.2	139.8	10,000
HCF GO	935.2	139.8	10,000
HDF HGO	935.2	139.8	10,000
CRS LR	741.3	116.7	10,000
RFO	547.5	72.2	10,000
HA	940.2	142.00	10,000

The blending component's properties are illustrated in Table 4. The specifications of diesel include flash point, sulfur content, viscosity, D90 and density. Fuel oil is specified in flash point, sulfur, viscosity and density. The units of naphthalene, flash point, smoke point, freezing point, sulfur, viscosity, D90 and density are %volume, ℃, millimeters, ℃, ppm, cSt, ℃ and km/m^3, respectively. The intermediate streams must be blended to be within the product specifications in Table 5.

Table 4a Component properties of HDF Kero, HCF Kero, KMU Kero, DHDS GO and HCF GO

Property	Unit	Blending Components								
		HDF Kero	HCF Kero	KMU Kero	DHDS GO	HCF GO	RFO	CRS LR	DHDS GO	HA
Naphthalene	%vol	2.2	0.5	1.46	-	-	-	-	-	-
Flash point	ºC	37.1	36.3	52.839	91	85	77	124	105	76
Smoke point	mm	23.6	30.9	22	-	-	-	-	-	-
Freezing point	ºC	-62	-60	-53	-	-	-	-	-	-
Conductivity		260	420	262	-	-	-	-	-	-
Sulfur	ppm	26.4	1.9	680	9.6	1	21200	1780	600	80
Viscosity	cSt	3	3	-	2.5	4.118	75.19	8.661	3.966	1.754
D90	ºC	225.6	214.3	-	358.7	356.1	-	-	-	-
Density	km/m^3	793	790	790.8	838.6	829	997.2	832.3	839.	952.4

Table 5 Product specifications

Property	Unit	Products				
		Jet Fuel	Diesel	Fuel Oil No.1	Fuel Oil No.2	Fuel Oil No.5
Naphthalene (Max)	%vol	3	-	-	-	-
Flash point (Min)	ºC	38	66	63	64	64
Smoke point (Min)	mm	19	-	-	-	-
Freezing point (Max)	ºC	-47	-	-	-	-
Conductivity		260 - 600	-	-	-	-
Sulfur (Max)	ppm	1500	50	19500	19500	94500
Viscosity	cSt	-	1.8 -4.1	25 -76	130- 176	290 -372
D90 (Max)	ºC	-	357	-	-	-
Density	km/m^3	775 -840	820 -870	930 - 980	935 - 985	935 - 985

We started running the procedure using two intervals and we found that even though there was some bound contraction, the solution of the bounds showed a 71.28% gap, requiring then an increase in the number of intervals. Noticing that the problem runs fast when increasing the number of intervals, we decided to compute the gap at iteration zero and do not continue with bound contraction. The objective functions and the gap are shown in Table 6 for different number of intervals. With 190 intervals, the relative error between the objective function of the UB and LB is 0.95% (less than 1%). The results of the profit and the recipe are shown in Table 7 ($42,555.142). We also run the problem using Antigone and Baron and obtained the same objective function value. The blended properties obtained by Rysia, Baron and Antigone are the same for all products except for Jet fuel and the recipes are different (Tables 8 and 9). The main reason is the flexibility in selecting the intermediate streams with similar quality specifications. Viscosity is the significant property to divide fuel oil grades. RFO is the most usage component of each fuel oil grade because of its low viscosity and lower cost than other intermediate streams of similar specification. The limiting constraint in diesel is the D90 value. HCF GO is the largest volume fraction for diesel because it is the cheapest diesel component and its D90 property is in the range of diesel specification. There are the limited choices for selecting intermediate streams to blend diesel and fuel oils; however, it is not the case for jet fuel. Considering Tables 4 and 5, all the properties of intermediate streams for jet, i.e. naphthalene, conductivity, freezing point, smoke point and sulfur, are within the jet fuel specification except flash point where HDF Kero and HCF Kero have lower flash point than the specification. These are shown in Table 9 and 10. Finally, Table 11, compares the three results.

As per solution time, the computational times are: 0.44 sec and 0.42 CPU sec for ANTIGONE and BARON, respectively. In turn, CONOPT, which is a local solver, gave an infeasible solution. Finally, RYSIA took 0.72 CPU seconds for the upper bound (190 intervals) and 0.8 CPU seconds for the lower bound, for a total of 1.6 CPU seconds. All these small differences in computational time are not fair comparisons. Indeed, while Baron and Antigone have been optimized for a number of years and are now in a form of automated compilation, also optimized, and have pre-solving routines, Rysia just makes use of GAMS input/output and is not yet optimized.

Table 6. Solution of the UB and LB models after increasing the number of intervals

Number of intervals	Upper bound	Lower bound	Relative error
2	72,888.809	42,555.142	71.28%
10	67,481.368	42,555.142	58.57%
40	48,359.599	42,555.142	13.64%
90	45,460.612	42,555.142	6.83%
120	45,220.038	42,555.142	6.26%
140	44,288.550	42,555.142	4.07%
170	43,982.785	42,555.142	3.35%
190	42,960.088	42,555.142	0.95%

Table 7. Blending recipe from Rysia (in m^3/h)

Profit	$42,555.142				
Product	Jet	Diesel	Fuel Oil No.1	Fuel Oil No.2	Fuel Oil No.5
HDF Kero	12,610.340	-	-	-	-
KMU Kero	3,484.527	-	-	-	-
HCF Kero	905.132	275.791	701.593	378.777	226.642
DHDS GO	-	2,661.483	-	-	-
HCF GO	-	12,062.726	-	-	-
RFO	-	-	3,286.640	2,645.308	4,096.131
HA	-	-	208.664	32.390	177.227
CRS LR	-	-	1,303.104	443.526	-

Table 8. Blending recipe from Baron (in m^3/h)

Profit	$42,555.142				
Product	Jet	Diesel	Fuel Oil No.1	Fuel Oil No.2	Fuel Oil No.5
HDF Kero	4,839.114	-	-	-	-
KMU Kero	5,826.689	-	-	-	-
HCF Kero	6,334.197	275.791	701.593	378.777	226.642
DHDS GO	-	2,661.483	-	-	-
HCF GO	-	12,062.726	-	-	-
RFO	-	-	3,286.640	2,645.308	4,096.131
HA	-	-	208.664	32.390	177.227
CRS LR	-	-	1,303.104	443.526	-

Table 9. Blending recipe from Antigone (in m^3/h)

Profit	$42,555.142				
Product	Jet	Diesel	Fuel Oil No.1	Fuel Oil No.2	Fuel Oil No.5
HDF Kero	-	-	-	-	-
KMU Kero	6,234.910	-	-	-	-
HCF Kero	10,765.090	275.791	701.593	378.777	226.642
DHDS GO	-	2,661.483	-	-	-
HCF GO	-	12,062.726	-	-	-
RFO	-	-	3,286.640	2,645.308	4,096.131
HA	-	-	208.664	32.390	177.227
CRS LR	-	-	1,303.104	443.526	-

Table 10. Blended properties from Rysia (in m^3/h)

Property	Jet fuel	Diesel	Fuel oil No.1	Fuel oil No.2	Fuel oil No.5
Napthalene	1.966	-	-	-	-
Conductivity	268.929	-	-	-	-
Smoke point index	0.042	-	-	-	-
Smoke point	23.763	-	-	-	-
Freezing point index	1.081	-	-	-	-
Freezing point	-59.941	-	-	-	-
Flash point index	704.765	47.984	147.341	137.951	98.968
Flash point	39.232	80.037	63	64	69.043
Viscosity index	-	11.391	28.428	30.983	32.941
Viscosity	-	3.63	76	176	372
D90 index	-	69064.767	-	-	-
D90	-	357	-	-	-
Sulfur content	158.828	2.558	13964.684	16955.788	19539.503
Density	792.430	829.986	930	953.465	985

Table 11. Comparison result of Rysia, Baron and Antigone (in m^3/h) for Jet Fuel.

Jet Fuel	**Rysia**	**Baron**	**Antigone**
Napthalene	1.966	1.327	0.867
Conductivity	268.929	320.301	362.052
Smoke point index	0.042	0.039	0.037
Smoke point	23.763	25.444	27.198
Freezing point index	1.081	1.097	1.103
Freezing point	-59.941	-58.052	-57.341
Flash point index	704.765	648.314	648.314
Flash point	39.232	40.5	40.500
Sulfur content	158.828	241.248	250.799
Density	792.430	791.197	790.367

CONCLUSIONS

We have shown that heavy product blending can be solved to global optimality using RYSIA (Faria and Bagajewicz 2011, Faria and Bagajewicz 2012, Kim and Bagajewicz, 2016). We presented a relaxed MILP model and we found that RYSIA solves very quickly for a large number of partitions. We also compared with Baron and Antigone and we found Rysia to spend similar time, although Rysia is not optimized to handle pre-processing, compilation and input/output.

NOMENCLATURE

Parameters

vT_p	total volume of oil product *p*
cp_s	capacity of intermediate stream *s*
nap_s	naphthalene volume percent of intermediate stream s
den_s	density of intermediate stream s
$cond_s$	conductivity of intermediate stream s
fpi_s	flash point index of intermediate stream s
spi_s	smoke point index of intermediate stream s
$frzi_s$	freezing point index of intermediate stream s
$d90i_s$	distillation 90% recovery index of intermediate stream s
$v50_s$	viscosity index of intermediate stream s
$d90coeff$	distillation 90% recovery coefficient
$temp_p$	temperature of oil product p
pr_p	price of oil product p
ct_s	cost of intermediate stream s
$fpiat_f$	flash point index value at the starting of interval f
$lfpiat_f$	logarithm of flash point index value at the starting of interval f
$d90iat_f$	distillation 90% recovery index value at the starting of interval f
$ld90iat_f$	logarithm of distillation 90% recovery index value at starting of interval f
$v50at_f$	viscosity index value at the starting of interval f

$visat_f$	viscosity value at starting of interval f
$sulat_f$	sulfur content value at starting of interval f
$wtat_f$	total weight value at starting of interval f
$spat_f$	smoke point value at starting of interval f
$frzat_f$	freezing point value at starting of interval f

Variables

WT_p	total weight of oil product p
$W_{s,p}$	intermediate stream s weight producing oil product p
$V_{s,p}$	intermediate stream s volume producing oil product p
NAP_p	naphthalene volume percent of oil product p
DEN_p	density of oil product p
$COND_p$	conductivity of oil product p
FP_p	flash point of oil product p
FPI_p	flash point index of oil product p
$V50_p$	viscosity index of oil product p
SUL_p	sulfur content of oil product p
SP_p	smoke point of oil product p
SPI_p	smoke point index of oil product p
FRZ_p	freezing point of oil product p
$FRZI_p$	freezing point index of oil product p
$D90_p$	distillation 90% recovery of oil product *p*
$D90I_p$	distillation 90% recovery index of oil product *p*
VIS_p	viscosity of oil product *p*
$V50_p$	viscosity index of oil product *p*
$PROFIT$	profit (objective function)
$LFPI_p$	logarithm of flash point index of oil product *p*
$LD90I_f$	logarithm of distillation 90% recovery index
$ZSUL_p$	the multiplication of sulfur content and total weight of oil product p
WWT_f	the multiplication of binary variable of sulfur and total weight of oil product

$ZV50_p$ the multiplication of viscosity index and total weight of oil product p

$WV50_f$ the multiplication of binary variable of total weight and viscosity index of oil product

$WSPI_f$ the product of binary variable of smoke point and smoke point index of oil product

$ZFRZ_p$ the product of freezing point and freezing point index of oil product *p*

$WFRZ_f$ the product of binary variable of freezing point and freezing point index

$yFPI_f$ binary variable for flash point index at interval *f*

$yD90I_f$ binary variable for distillation 90% recovery index at interval *f*

$yV50_f$ binary variable for viscosity index at interval *f*

$ySUL_f$ binary variable for sulfur at interval *f*

yWT_f binary variable for total weight at interval f

ySP_f binary variable for smoke point at interval f

$OMEGA$ Parameter used in big-M constraints.

REFERENCES

Chen, X. and N. Wang (2010). "Optimization of short-time gasoline blending scheduling problem with a DNA based hybrid genetic algorithm." Chemical Engineering and Processing: Process Intensification **49**(10): 1076-1083.

Faria, D. C. and M. J. Bagajewicz (2011). "Novel bound contraction procedure for global optimization of bilinear MINLP problems with applications to water management problems." Computers & Chemical Engineering **35**(3): 446-455.

Faria, D. C. and M. J. Bagajewicz (2012). "A new approach for global optimization of a class of MINLP problems with applications to water management and pooling problems." Aiche Journal **58**(8): 2320-2335.

Faria D., S. Y. Kim and M. Bagajewicz. *Global Optimization of the Stage-wise Superstructure Model for Heat Exchanger Networks.* Industrial and Engineering Chemistry Research. .Vol. 54 No 5, pp 1595–1604 (2015).

Gary, J. H. and G. E. Handwerk (2007). Petroleum Refining: Technology and Economics, . New York, Marcel Dekker.

Ierapetritou, M. G.; Floudas, C. A. Short-term Scheduling: New Mathematical Models vs. Algorithmic Improvements. *Computers and Chemical Engineering* **1998,** *22,* S419-S426.

Ierapetritou, M. G.; Floudas, C. A. Effective Continuous-Time Formulation for Short-Term Scheduling. 2. Continuous and Semicontinuous Processes. *Industrial & Engineering Chemistry Research* **1998,** *37*, 4360-4374.

Ierapetritou, M. G.; Hene, T. S.; Floudas, C. A. Effective Continuous-Time Formulation for Short Term Scheduling. 3. Multiple Intermediate Due Dates. *Industrial & Engineering Chemistry Research* **1999,** *38* (9), 3446-3461.

Jia, Z. Y. and M. Ierapetritou (2003). "Mixed-integer linear programming model for gasoline blending and distribution scheduling." Industrial & Engineering Chemistry Research **42**(4): 825-835.

Kim S. Y. and M. Bagajewicz. (2016). *Global Optimization for Heat Exchanger Network Design using a Generalized Superstructure*. CES. (available online: 9-FEB-2016 DOI information: 10.1016/j.ces.2016.02.002).

Lakkhanawat H. and M. Bagajewicz. (2008). *Financial Risk Management with Product Pricing in the Planning of Refinery Operations*. Industrial and Engineering Chemistry Research. Vol 47, No 17, 6622-6639.

Mendez, C. A., I. E. Grossmann, I. Harjunkoski and P. Kabore (2006). "A simultaneous optimization approach for off-line blending and scheduling of oil-refinery operations." Computers & Chemical Engineering **30**(4): 614-634.

Meyer, C. A. and C. A. Floudas (2006). "Global optimization of a combinatorially complex generalized pooling problem." AIChE Journal **52**(3): 1027-1037.

Misener, R.; Gounaris, C. E.; Floudas, C. A. Mathematical modeling and global optimization of large scale extended pooling problems with the EPA complex emissions contraints. *Computers and Chemical Engineering* **2010,** *34*, 1432-1456.

Moro, L. F. L., A. C. Zanin and J. M. Pinto (1998). "A planning model for refinery diesel production." Computers and Chemical Engineering **22**(SUPPL.1): S1039-S1042.

Murty, B. S. N. and R. N. Rao (2004). "Global optimization for prediction of blend composition of gasolines of desired octane number and properties." Fuel Processing Technology **85**(14): 1595-1602.

Pongsakdi A., K. Siemanond, P. Rangsunvigit and M. Bagajewicz (2006). *Financial Risk Management in Refinery Operations Planning under Uncertainty.* International Journal of Production Economics, 103, 1, 64-86.